Relativistic contraction of an accelerated rod

Hrvoje Nikolić
Theoretical Physics Division, Rudjer Bošković Institute,
P.O.B. 1016, HR-10001 Zagreb, Croatia
hrvoje@faust.irb.hr

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Abstract

The relativistic motion of an arbitrary point of an accelerated rigid rod is discussed for the case when velocity and acceleration are directed along the rod's length. The discussion includes the case of a time-dependent force applied to a single point, as well as a time-independent force arbitrary distributed along the rod. The time dependence of the rod's relativistic length depends on the application point of the force, but after the termination of acceleration the final velocity and length do not depend on it. An observer on a uniformly accelerated rod feels an inertial force which decreases in the direction of acceleration. The influence of non-rigidity of realistic rods on our results is qualitatively discussed.

1 Introduction

There are several articles which discuss relativistic properties of accelerated rods for the case when the force is time-independent and applied to a single point on the rod. Cavalleri and Spinelli [1] found two important results for such a case. First, the application point accelerates in the same way as it would accelerate if all mass of the rod were concentrated in this point. Second, "a rod, pushed by a force, accelerates less than the same rod pulled by the same force". Some similar results were found by Nordtvedt [2] and Grøn [3], who concluded that "a rocket ship can be accelerated to higher speeds with a given engine by putting the engine in the nose of the rocket". We agree with the first statement in quotation marks, but we disagree with the second one. At first sight, the second statement in quotation marks may seem to be a consequence of the first one. On the other hand, the second statement cannot be consistent with the conservation of energy. We resolve the paradox by generalizing the analysis to time-dependent forces. As an example we consider the case of a uniformly accelerated rod during a finite time interval, after which the force turns off. It appears that although the motion of the rod depends on the application point of the force, the final velocity and relativistic length after the termination of acceleration do not depend on it.

From the point of view of an inertial observer at rest, a pushed rod accelerates slower than a pulled one, but the acceleration of a pushed rod lasts longer than that of a pulled one.

In this article we also generalize the first result of [1] by considering the case of many time-independent forces applied to various points. There is one point which accelerates in the same way as it would accelerate if all forces were applied to this point. We find that this point is given by (35).

The paper is organized as follows: In Section 2 we present a general method of determining the motion of an arbitrary point of a non-rotating rigid rod, when the motion of one of its points is known. We also present a general method of determining the motion of a point on the rigid rod, to which a single time-dependent force is applied. In Section 3 we apply this general formalism to the case of a uniformly accelerated rod during a finite time interval, after which the force turns off. In Section 4 we discuss the physical meaning of the results obtained in Section 3. In Section 5 we analyze the case of many time-independent forces applied to various points. In Section 6 we give a qualitative discussion of how the non-rigidity of realistic rods alters our analysis and find conditions under which our analysis is still valid, at least approximately. Section 7 is devoted to concluding remarks.

2 The length of a rigid accelerated rod

Let us consider a rod whose velocity and acceleration are directed along its length, but are otherwise arbitrary functions of time. We assume that the accelerated rod is rigid, which means that an observer located on the rod does not observe any change of the rod's length. (In Section 6 we discuss the validity of such an assumption.) Since the rod is rigid and does not rotate, it is enough to know how one particular point of the rod (labeled for example by A) changes its position with time. Let S be a stationary inertial frame and S' the accelerated frame of an observer on the rod. We assume that we know the function $x_A(t_A)$, so we also know the velocity

$$v(t_A) = \frac{dx_A(t_A)}{dt_A} \; . \tag{1}$$

Later we consider how the function $x_A(t_A)$ can be found if it is known how the forces $F(t')$ act on the rod. The function (1) defines the infinite sequence of comoving inertial frames $S'(t_A)$. The rod is instantaneously at rest for an observer in this frame. This means that he observes no contraction, i.e., $L_0 = x'_B - x'_A$, where L_0 is the proper length of the rod, while A and B label the back and front ends of the rod, respectively. He observes both ends at the same instant, so $t'_B - t'_A = 0$. From the Lorentz transformations

$$x_{A,B} = \gamma_v(t_A)(x'_{A,B} + v(t_A)t'_{A,B}) \; , \qquad t_{A,B} = \gamma_v(t_A)\left(t'_{A,B} + \frac{v(t_A)}{c^2}x'_{A,B}\right) \; , \tag{2}$$

where $\gamma_v(t_A) = (1 - v^2(t_A)/c^2)^{-1/2}$, we obtain

$$x_B - x_A = L_0\gamma_v(t_A) \; , \tag{3}$$

$$t_B - t_A = L_0\gamma_v(t_A)\frac{v(t_A)}{c^2} \; . \tag{4}$$

From (3) and the known functions $x_A(t_A)$ and (1) we can find the function $x_B(t_A)$. From (4) and the known function (1) we can find the function $t_A(t_B)$. Thus we find the function

$$x_B(t_A(t_B)) \equiv \tilde{x}_B(t_B) \; . \tag{5}$$

To determine how the rod's length changes with time for an observer in S, both ends of the rod must be observed at the same instant, so $t_B = t_A \equiv t$. Thus the length as a function of time is given by

$$L(t) = \tilde{x}_B(t) - x_A(t) \; . \tag{6}$$

Let us now see how velocity (1) can be found if the force $F(t'_A)$ applied to the point A is known. F is the force as seen by an observer in S'. We introduce the quantity

$$a(t'_A) = F(t'_A)/m(t'_A) \; , \tag{7}$$

which we call acceleration, having in mind that this would be the second time derivative of a position only in the nonrelativistic limit. Here $m(t'_A)$ is the proper mass of the rod, which, in general, can also change with time, for example by loosing the fuel of a rocket engine. As shown in [1], if there is only one force, applied to a specific point on an elastic body, and if F and m do not vary with time, then this point moves in the same way as it would move if all mass of the body were concentrated in this point. If acceleration changes with time slowly enough, then this is approximately true for a time-dependent acceleration as well. Later we discuss the conditions for validity of such an approximation. Here we assume that these conditions are fulfilled. The application point is labeled by A. Thus, by a straightforward application of the velocity addition formula, we find that the infinitesimal change of velocity is given by

$$u(t'_A + dt'_A) = \frac{u(t'_A) + a(t'_A)dt'_A}{1 + \dfrac{u(t'_A)a(t'_A)dt'_A}{c^2}} = u(t'_A) + \left(1 - \frac{u^2(t'_A)}{c^2}\right) a(t'_A)dt'_A \; , \tag{8}$$

where $u(t'_A)$ is velocity defined in such a way that $u(t'_A(t_A)) = v(t_A)$. Since $u(t'_A + dt'_A) = u(t'_A) + du$, this leads to the differential equation

$$\frac{du(t'_A)}{dt'_A} = \left(1 - \frac{u^2(t'_A)}{c^2}\right) a(t'_A) \; , \tag{9}$$

which can be easily integrated, since $a(t'_A)$ is the known function by assumption. Thus we find the function $u(t'_A)$. To find the function $v(t_A)$, we must find the function $t'_A(t_A)$. We find this from the infinitesimal Lorentz transformation

$$dt_A = \frac{dt'_A + \dfrac{u(t'_A)}{c^2}dx'_A}{\sqrt{1 - u^2(t'_A)/c^2}} \; . \tag{10}$$

The point on the rod labeled by A does not change, i.e., $dx'_A = 0$, so (10) can be integrated as

$$t_A = \int \frac{dt'_A}{\sqrt{1 - u^2(t'_A)/c^2}} \; , \tag{11}$$

which gives a function $t_A = f(t'_A)$ and thus $t'_A = f^{-1}(t_A)$.

It is also interesting to see how the length of an unaccelerated rod changes with time from the point of view of an accelerated observer. The generalized Lorentz transformations between an inertial frame and an accelerated frame, as shown by Nelson [4], are given by

$$x = \gamma_u(t')x' + \int_0^{t'} \gamma_u(t')u(t')dt' \ , \qquad t = \frac{\gamma_u(t')u(t')}{c^2}x' + \int_0^{t'} \gamma_u(t')dt' \ , \tag{12}$$

where $\gamma_u(t') = (1 - u^2(t')/c^2)^{-1/2}$. Now the proper length of the rod is $L_0 = x_2 - x_1$. The accelerated observer observes both ends at the same instant, so $t'_2 = t'_1$. The length that he observes is $L' = x'_2 - x'_1$, so from (12) we find

$$L'(t') = \frac{L_0}{\gamma_u(t')} \ . \tag{13}$$

3 Uniformly accelerated rod during a finite time interval

In the preceding section we have made a very general analysis. Here we want to illustrate these results on a simple realistic example, in order to understand the physical meaning of these general results. We consider the case of a rod which is at rest for $t < 0$, but at $t = 0$ it turns on its engine which gives the constant acceleration a to the application point during a finite time interval T', after which the engine turns off. From (9) and (11) for $t'_A < T'$ we find

$$u(t'_A) = c \operatorname{tgh}\frac{at'_A}{c} \ , \tag{14}$$

$$t_A(t'_A) = \frac{c}{a} \operatorname{sh}\frac{at'_A}{c} \ , \tag{15}$$

and thus

$$v_A(t_A) = \begin{cases} \dfrac{at_A}{\sqrt{1+(at_A/c)^2}} \ , & 0 \leq t_A \leq T \ , \\[2ex] \dfrac{aT}{\sqrt{1+(aT/c)^2}} \ , & t_A \geq T \ , \end{cases} \tag{16}$$

where

$$T = \frac{c}{a} \operatorname{sh}\frac{aT'}{c} \ . \tag{17}$$

With the initial condition $x_A(t_A = 0) = 0$ we obtain

$$x_A(t_A) = \begin{cases} \sqrt{(c^2/a)^2 + (ct_A)^2} - c^2/a \ , & 0 \leq t_A \leq T \ , \\[2ex] \dfrac{aTt_A}{\sqrt{1+(aT/c)^2}} + \dfrac{c^2}{a}\left(\dfrac{1}{\sqrt{1+(aT/c)^2}} - 1\right) \ , & t_A \geq T \ . \end{cases} \tag{18}$$

The rest of job is described by the procedure given from (3) to (6). Thus we find

$$t_A(t_B) = \begin{cases} \dfrac{t_B}{1 + aL_0/c^2} \; , & 0 \le t_B \le T_+ \; , \\ \\ t_B - aL_0T/c^2 \; , & t_B \ge T_+ \; , \end{cases} \tag{19}$$

$$\tilde{x}_B(t_B) = \begin{cases} \sqrt{1 + \dfrac{(at_B/c)^2}{(1+aL_0/c^2)^2}} \left(\dfrac{c^2}{a} + L_0 \right) - \dfrac{c^2}{a} \; , & 0 \le t_B \le T_+ \; , \\ \\ \dfrac{1}{\sqrt{1+(aT/c)^2}} \left(\dfrac{c^2}{a} + L_0 + aTt_B \right) - \dfrac{c^2}{a} \; , & t_B \ge T_+ \; , \end{cases} \tag{20}$$

$$L(t) = \begin{cases} \sqrt{1 + \dfrac{(at/c)^2}{(1+aL_0/c^2)^2}} \left(\dfrac{c^2}{a} + L_0 \right) - \dfrac{c^2}{a} \sqrt{1+(at/c)^2} \; , \quad 0 \le t \le T \; , \\ \\ \sqrt{1 + \dfrac{(at/c)^2}{(1+aL_0/c^2)^2}} \left(\dfrac{c^2}{a} + L_0 \right) - \dfrac{1}{\sqrt{1+(aT/c)^2}} \left(\dfrac{c^2}{a} + aTt \right) \; , \\ \qquad\qquad\qquad\qquad\qquad\qquad T \le t \le T_+ \; , \\ L_f \; , \quad t \ge T_+ \; , \end{cases} \tag{21}$$

where $T_\pm = T(1 \pm aL_0/c^2)$, while $L_f = L_0/\sqrt{1+(aT/c)^2}$ is the final length. Note that (21) differs from the result which one could expect from the naive generalization of the Lorentz-Fitzgerald formula

$$L(t) = L_0\sqrt{1 - v^2(t)/c^2} = \frac{L_0}{\sqrt{1+(at/c)^2}} \; , \quad 0 \le t \le T \; . \tag{22}$$

Formula (21) was obtained for the case when the force is applied to the back end of the rod. In other words, this is the result for a pushed rod. The analysis for a pulled rod is similar and the result is

$$L(t) = \begin{cases} \dfrac{c^2}{a}\sqrt{1+(at/c)^2} - \sqrt{1 + \dfrac{(at/c)^2}{(1-aL_0/c^2)^2}} \left(\dfrac{c^2}{a} - L_0 \right) \; , & 0 \le t \le T_- \; , \\ \\ \dfrac{c^2}{a}\sqrt{1+(at/c)^2} - \dfrac{1}{\sqrt{1+(aT/c)^2}} \left(\dfrac{c^2}{a} - L_0 + aTt \right) \; , & T_- \le t \le T \; , \\ L_f \; , \quad t \ge T \; . \end{cases} \tag{23}$$

Finally, let us see how an unaccelerated rod looks from the point of view of an accelerated observer. From (13), (14) and (17) we find

$$L'(t') = \begin{cases} \dfrac{L_0}{\sqrt{1 + \mathrm{sh}^2 at'/c}} \; , & 0 \le t' \le T' \; , \\ \\ L_f \; , \quad t' \ge T' \; . \end{cases} \tag{24}$$

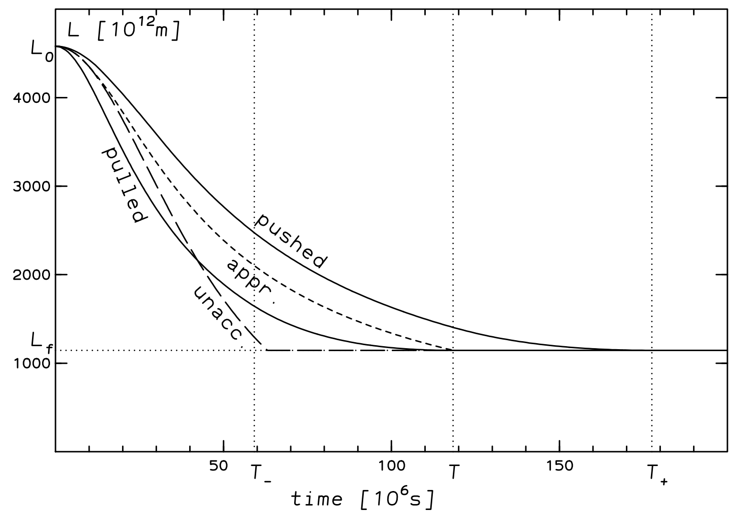

Figure 1: The rod's length as a function of time. The solid curves represent the lengths of the pushed and the pulled rods (21) and (23), respectively. The short-dashed curve represent the approximative result (22), while the long-dashed curve represent the length of an unaccelerated rod (24).

The results (21), (23), (22) and (24) are depicted in Fig.1. The parameters are chosen such that $a = 9.81\ \mathrm{ms}^{-2}$, $aL_0/c^2 = 0.5$, $\sqrt{1 + (aT/c)^2} = 4$, not with the intention to represent a realistic case, but rather to obtain results which will provide a synoptic view of the figure. The solid curves represent the lengths of the pushed and the pulled rods (21) and (23), respectively. The short-dashed curve represents the approximative result (22), while the long-dashed curve represents the length of an unaccelerated rod (24).

4 Interpretation

We see the inequivalence between an unaccelerated observer observing an accelerated rod and an accelerated observer observing an unaccelerated rod. For an accelerated observer, the acceleration lasts for a shorter time, as can be seen from (17). The time dependence of the length of an unaccelerated rod is given by (13), which can be understood as a simple generalization of the well-known Lorentz-Fitzgerald formula. On the other hand, a similar simple generalization (22) does not work for an accelerated rod. However, (22) is a good approximation if $aL_0/c^2 \ll 1$.

We also see that the results for a pulled rod are meaningless if $aL_0/c^2 > 1$. This suggests that the rod cannot remain rigid under such conditions. To understand why is that so, we calculate the velocity of the back end for a pulled rod. The result is

$$\frac{d\tilde{x}_A(t)}{dt} = \frac{at}{\sqrt{(1 - aL_0/c^2)^2 + (at/c)^2}} \, . \tag{25}$$

We see that this velocity increases as aL_0 increases and reaches the velocity of light when $aL_0 = c^2$. Since no part of the rod can exceed the velocity of light, the rod cannot remain rigid for $aL_0/c^2 > 1$. This is true for a pushed rod as well, as will be more clear from the discussion of Section 6.

Although the time dependence of the rod's length depends on whether the rod is pushed, pulled or unaccelerated, the final length and velocity after the forces are turned off do not depend on it. But what varies is the time needed to observe the termination of acceleration. An unaccelerated observer observes that the acceleration of the front end of the accelerated rod lasts longer than that of the back end. If the rod is pulled, it seems to him that the consequence (termination of acceleration of the back end) occurs before the cause (termination of acceleration of the front end). However, this is not a paradox, because there is a spacelike separation between these two events, which is an artefact of the rigidity assumption. We make more comments on this in Section 6. An unaccelerated observer cannot actually know whether a rigid rod is pulled by an acceleration a, or is pushed by an acceleration $\tilde{a}$, given by

$$\tilde{a} = \frac{a}{1 - aL_0/c^2} \ . \tag{26}$$

This can be seen for example by replacing the acceleration in the first case ($0 \leq t \leq T$) in (21) by $\tilde{a}$ and comparing it with the first case ($0 \leq t \leq T_-$) in (23). In particular, if the rod is pulled by acceleration $a = c^2/L_0$, for an unaccelerated observer it looks the same as it is pushed by acceleration $\tilde{a} = \infty$. If this pulling lasts time T, this is the same as the pushing lasts time $T_- = 0$.

Formula (26) can be generalized to an arbitrary point on the rod. If acceleration a is applied to the point x'_A, then this is the same as acceleration $a(x')$ is applied to the point x', where

$$a(x') = \frac{a}{1 + (x' - x'_A)a/c^2} \ . \tag{27}$$

The important consequence of this is that an observer in a uniformly accelerated rocket does *not* feel a *homogeneous* inertial force, but rather an inertial force which decreases with x', as given by (27). This result is obtained also in [3]. A long rigid rod is equivalent to a series of independent shorter rods, each having its own engine, but not with equal accelerations, but rather with accelerations which are related by formula (27).

Note also that one can replace a by $a(x')$ in the first case ($0 \leq t_A \leq T$) in (16) and thus obtain how the velocity of various points of a rod depends on time. The result coincides with the result obtained by Cavalleri and Spinelli [1]. They found this result by solving a certain partial differential equation, so our derivation is much simpler. In addition, our method allows a generalization to time-dependent accelerations as well.

5 A set of forces with various application points

It is shown in [1] that if a time-independent force is applied to a *single* point on the rod, then this point moves in the same way as it would move if all mass of the rod were concentrated in this point. However, if there are many forces directed along the length of a rigid rod, each applied to a different point on the rod, then, obviously, all these points cannot move in

the same way as they would move if all mass of the rod were concentrated in these points. We remind the reader that it is enough only to find out how one particular point of the rod moves, because the motion of the rest of rod is determined by the rigidity requirement. Thus the problem of many forces can be reduced to a problem of finding a point x which moves in the same way as it would move if all forces were applied to this point (in this section we omit a prime on x', remembering that this is a coordinate on the rod in the accelerating rod's frame).

Assume that N forces F_i, $i = 1, \ldots, N$, are applied to the rod, each applied to the point x_i. If all forces are of the same sign, then the rod (with a finite width) can be cut in N pieces, each with a mass m_i and each with only one applied force F_i, in such a way that the collection of pieces moves in the same way as the whole rod would move without the cutting. The masses of the pieces satisfy

$$\sum_i m_i = m \; , \tag{28}$$

where m is the mass of the whole rod. We also introduce the notation

$$a_i = \frac{F_i}{m_i} \; , \quad a = \frac{F}{m} \; , \tag{29}$$

where $F = \sum_i F_i$. From (27) it follows

$$\frac{c^2}{a_i} - \frac{c^2}{a_j} = x_i - x_j \; , \tag{30}$$

which leads to $N - 1$ independent equations

$$c^2 \frac{m_{i+1}}{F_{i+1}} - c^2 \frac{m_i}{F_i} = x_{i+1} - x_i \; , \quad i = 1, \ldots, N-1 \; . \tag{31}$$

This, together with (28), makes a system of N independent equations for N unknown masses m_i, with the unique solution

$$m_i = \frac{F_i}{\sum_j F_j} \left[m - \frac{1}{c^2} \sum_k F_k (x_k - x_i) \right] \; . \tag{32}$$

However, the masses m_i are only auxiliary quantities. From (32) and the first equation in (29) we find

$$a_i = \frac{\sum_j F_j}{m - \frac{1}{c^2} \sum_k F_k (x_k - x_i)} \; . \tag{33}$$

This is one of the final results, where masses m_i do not appear. When $N = 1$ or all forces are applied to the same point, (33) reduces to the already known result that the application point has the acceleration $a_i = F/m$.

There is one point which accelerates in the same way as it would accelerate if all forces were applied to this point. This point x is given by

$$\frac{c^2}{a} - \frac{c^2}{a_i} = x - x_i \; , \tag{34}$$

so from (34), (33) and the second equation in (29) we find

$$x = \frac{\sum_k F_k x_k}{\sum_j F_j} \ . \tag{35}$$

Formulas (33) and (35) are the main new results of this section. We have derived them under the assumption that all forces F_i are of the same sign. However, we believe that this assumption is not crucial for the validity of (33) and (35). Since there must exist general formulas which reduce to (33) and (35) when all forces are of the same sign, we conjecture that these general formulas can be nothing else but (33) and (35) themselves. For example, one could suspect that, in a general formula, F_k should be replaced by $|F_k|$, but one can discard such a possibility by considering the case when some forces of the different signs are applied to the same point.

Are there any problems if all forces are not of the same sign? We assume that the square bracket in (32) is always positive (see the discussion connected with formula (25)). Thus, if some force F_i is of the opposite sign with respect to the total force $F = \sum_j F_j$, then the corresponding mass m_i is formally negative, which means that the rod cannot be cut in a way which was described at the beginning of this section. However, we believe that cutting the rod is not essential at all.

From (35) one can also see that if all forces are not of the same sign, then the point x may not lie on the rod itself. This may look slightly peculiar, but is not inconsistent.

Formulas (33) and (35) can be easily generalized to a continuous distribution of force. For example, (35) generalizes to

$$x = \frac{\int dy \, f(y) y}{\int dy \, f(y)} \ , \tag{36}$$

where $f(y)$ is a linear density of force, $f(y) = dF/dy$.

6 Discussion

In this section we give a qualitative discussion of how the non-rigidity of realistic rods alters our analysis and find conditions under which our analysis is still valid, at least approximately.

First, it is clear that, in general, the proper length of a uniformly accelerated rod will not be equal to the proper length of the same rod when it is not accelerated. For example, we expect that a pushed rod will be contracted, while a pulled rod will be elongated. This is not a relativistic effect, but rather a real change of a proper length. It is important, however, that if acceleration does not change with time, then this proper length does not change with time either. Therefore, all formulas of this article which describe a uniform acceleration during a long time interval, are correct if L_0 is understood as a proper length which depends on the acceleration and application point of the force, but not on the time of the accelerated frame.

The dynamics of a rod when acceleration changes with time is more complicated. However, some qualitative conclusions can be drawn without much effort. When acceleration is changed, the rod needs some relaxation time Δt (here t is time in the rod's accelerated frame) to reach a new equilibrium proper length which depends on the new acceleration.

During this time we expect one or a few damped oscillations, so Δt is of the order

$$\Delta t \approx L_0/v_s \,, \tag{37}$$

where v_s is the velocity of propagation of a disturbance in a material. This velocity is equal to the velocity of sound (not of light) in a material of which is the rod made. If acceleration changes slowly enough, then we can use the adiabatic approximation, i.e., we can assume that the length of the rod is always equal to its equilibrium length which depends on the instantaneous acceleration. The small change of acceleration means that $\Delta a/a \ll 1$ during the relaxation time Δt, so from (37) and the relation $\Delta a = \dot{a}\Delta t$, we find the criteria for the validity of the adiabatic approximation

$$\dot{a} \ll \frac{a v_s}{L_0} \,. \tag{38}$$

In practice, $\dot{a}$ is never infinite, i.e., the instantaneous changes of acceleration do not exist.

As discussed in Section 4, owing to the rigidity assumption, it can happen that the consequence precedes the cause. In a more realistic calculation, which respects that v_s is not larger than c, this will not be the case. However, from the point of view of an unaccelerated inertial observer, it will still be true that the acceleration of a pushed rod lasts longer than that of a pulled one.

Let us now investigate the conditions under which a rod can be considered as approximately rigid, in the sense that the change of the proper length ΔL is much smaller than the proper length L_0 of the unaccelerated rod. If the force is applied to the front end of the rod, then after the time given by (37) the back end will receive the information that it also has to accelerate. Therefore, the change of the proper length is of the order $\Delta L \approx a(\Delta t)^2 \approx aL_0^2/v_s^2$. Since $v_s < c$, the requirement $\Delta L/L_0 \ll 1$ leads to the requirement

$$aL_0/c^2 \ll 1 \,. \tag{39}$$

In that case, (22) is a good approximation and the difference between the pushed and the pulled rod is negligible.

Finally, note that the results of Sections 2 and 3 are exact if labels A and B do not refer to fixed points on the rod, but one of the labels refers to the observer whose acceleration *is* given by $a(t')$ and the other refers to the point the distance of which from the first point *is* L_0, as seen by the observer at the first point.

7 Conclusion

In this article we have relativistically solved a general problem of motion of an arbitrary point of a rigid rod accelerated by a time-dependent force applied to a single point, for the case when the force and velocity are directed along the rod's length. The time-dependence of a rod's relativistic length depends on the application point of the force, but the final velocity and length after the termination of acceleration do not depend on it. An observer on a uniformly accelerated rod does not feel a homogeneous inertial force, but rather an inertial force which decreases in the direction of acceleration. Formula (35) determines the motion

of a rigid rod when many time-independent forces directed along the rod's length are applied to various points. The case of many time-dependent forces applied to various points is more complicated, so we have not considered this case. In addition, we have given a qualitative discussion of how the non-rigidity of realistic rods alters our analysis and found conditions under which our analysis is still valid, at least approximately.

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